Telegrapher's equation attenuation

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1 "Constant" attenuation derivation.

We modify the wave equation by adding a u_t term to get the telegrapher's equation:

$$u_{tt} - u_{xx} + u_t = 0. \tag{1}$$

We analyze the attenuation that is introduced by plugging in a plane wave of the form

$$u(x,t) = e^{i(kx-\omega t)}. \tag{2}$$

This gives dispersion relation

$$k = \pm\sqrt{\omega^2 + i\omega}. \tag{3}$$

For the wave equation, we have $k = \pm\omega \in \mathbb{R}$ and there is no attenuation. Now $k \in \mathbb{C}$ so there is attenuation. The attenuation comes from the imaginary part of k, so we want to look at how this depends on ω. We can write k^2 in polar form i.e.

$$k^2 = re^{i\theta} \tag{4}$$

where

$$r = \sqrt{\omega^4 + \omega^2} = \omega\sqrt{\omega^2 + 1} \tag{5}$$

and

$$\theta = \arctan(1/\omega). \tag{6}$$

Then we have that

$$k = \pm\sqrt{r}e^{i\theta/2} \tag{7}$$

and so

$$\Im(k) = \pm\sqrt{r}\sin(\theta/2) \tag{8}$$

For $\omega >> 1$, we have that $\arctan(1/\omega) << 1$ and therefore $\sin(\arctan(1/\omega)) \sim 1/\omega$, and so we have

$$\lim_{\omega\to\infty} \Im(k) = \lim_{\omega\to\infty} \omega\frac{1}{\omega} = 1. \tag{9}$$

So as ω becomes large, the rate of attenuation starts to look constant with respect to ω.